Robust Model Free Control of Robotic Manipulators with Prescribed Transient and Steady State Performance

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***Abstract*— In this paper, we propose a robust model free control scheme of minimal complexity (it is a static scheme involving very few and simple calculations to output the control signal) for robotic manipulators, capable of achieving prescribed transient and steady state performance. No information regarding the robot dynamic model is employed in the design procedure. Moreover, the tracking performance of the developed scheme (i.e., convergence rate and steady state error) is a priori and explicitly imposed by a designer-specified performance function, and is fully decoupled by both the control gains selection and the robot dynamic model. In that respect, the selection of the control gains is only confined to adopting those values that lead to reasonable control effort. Finally, two experimental studies in the joint and the Cartesian workspace clarify the design procedure and verify its performance and robustness against external disturbances.**

I. Introduction

During the last two decades, robotic manipulators have been used to perform difficult precision tasks in highly risky environments (e.g., assembly lines, exploration, surgery, etc.). In such robotic applications, where accuracy is of outmost importance, guaranteeing a desirable transient and steady state response of the closed loop system is a very significant issue. Moreover, the development of control schemes independent of the robot dynamic model increases their applicability (i.e., the same control scheme can be applied in any robotic manipulator without extensive and task-oriented modification) as well as their robustness against model uncertainties (e.g., modifying the robot end effector to perform a different task or attaching a device on the end effector to enhance sensing, alters the dynamic model).

PID controllers for robotic manipulators have been established since the early '90s, dealing, nevertheless, with the closed loop system stability problem rather than the performance issue [1]–[3]. Subsequently, in an attempt to improve the closed loop system performance, PID gain tuning procedures came up, which, however, required prior knowledge of the robot dynamic model [4]. Concurrently, employing the Lyapunov theory, the Slotine-Li adaptive controller [5] was established, without any transient performance guarantees, based on a complete robot model with parametric uncertainties. Moreover, a sliding mode controller with transient guarantees depending, however, on the control gains selection and some robot model constants, was proposed in [6]. Finally, based on the prescribed performance notion [7], theoretical results on PID type regulation controllers, with transient and steady state performance guarantees, were presented in [8], [9].

In this paper, extending the results in [10], we present a robust model free controller for robotic manipulators that achieves trajectory tracking with prescribed transient and steady state performance. The proposed scheme is of low complexity and does not require any information regarding the robot dynamic model. The tracking performance is imposed by a designer-specified performance function, and is decoupled by the control gains selection and the robot dynamic model, thus simplifying further the design procedure.

The paper is organized as follows. The problem formulation and some preliminary results, necessary in the subsequent analysis, are given in Section II. The control design procedure is provided in Section III. In Section IV the experimental results are illustrated and Section V describes a small accompanying video of the experiments. Finally, we conclude in Section VI.

II. Problem Formulation and Preliminaries

Consider an n degrees of freedom robotic manipulator and let $q=[q_1,\dots,q_n]^T\in\Re^n$ be the vector of the generalized joint variables. The robot dynamic model can be written in terms of coordinates q, as follows:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)+F_r(q,\dot{q})=\tau+D(t) \quad (1)$$

where $M(q)$ is the positive definite inertia matrix, $C(q,\dot{q})\dot{q}$ describes the centripetal and coriolis forces, $G(q)$ is the gravity vector, $F_r(q,\dot{q},t)$ represents unmodeled friction terms, $D(t)$ are bounded external disturbances and $\tau\in\Re^n$ denotes the applied torques at the joints. All aforementioned vector fields $M:\Re^n\to\Re^{n\times n}$, $C:\Re^n\times\Re^n\to\Re^{n\times n}$, $G:\Re^n\to\Re^n$, $F_r:\Re^n\times\Re^n\to\Re^n$ and $D:\Re_+\to\Re^n$ are continuous and there exist positive constants $\underline{m},\overline{m}>0$ such that:

$$\underline{m}I_n\leq M(q)\leq\overline{m}I_n,\ \forall q\in\Re^n, \quad (2)$$

where I_n denotes the $n\times n$ unitary matrix.

The objective of this paper is to design a control law τ, without incorporating any information regarding the robot dynamic model, that achieves tracking of a smooth and bounded desired trajectory $q_d(t)=[q_{d_1}(t),\dots,q_{d_n}(t)]^T\in\Re^n$ with a priori specified convergence rate and steady state error.

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A. Prescribed Performance

In this work, the control design is connected to the prescribed performance notion that was proposed to design robust state feedback controllers, for various classes of nonlinear systems [7], [10]–[13], capable of guaranteeing output tracking with prescribed performance. For completeness and compactness of presentation, this subsection summarizes preliminary knowledge on the concept of prescribed performance. Therefore, consider a generic scalar error $e(t)$. Prescribed performance is achieved if $e(t)$ evolves strictly within a predefined region that is bounded by decaying functions of time. The mathematical expression of prescribed performance is given, $\forall t \geq 0$, by the following inequalities:

$$-\rho(t) < e(t) < \rho(t) \tag{3}$$

where $\rho(t)$ is a smooth, bounded, strictly positive and decreasing function of time satisfying $\lim_{t\to\infty} \rho(t) > 0$, called performance function [7]. In this sense, consider an exponentially decreasing performance function $\rho(t) = (\rho_0 - \rho_\infty) e^{-lt} + \rho_\infty$ with ρ_0, ρ_∞, l appropriately chosen strictly positive constants. The constant $\rho_0 = \rho(0)$ is selected such that $\rho_0 > |e(0)|$. The constant $\rho_\infty = \lim_{t\to\infty} \rho(t)$ represents the maximum allowable size of the tracking error $e(t)$ at the steady state and can be set arbitrarily small to a value reflecting the resolution of the measurement device, thus achieving practical convergence of $e(t)$ to zero. Moreover, the decreasing rate of $\rho(t)$, which is affected by the constant l in this case, introduces a lower bound on the speed of convergence of $e(t)$.

B. Dynamical Systems

Consider the initial value problem:

$$\dot{\psi} = H(t,\psi),\ \psi(0) = \psi^0 \in \Omega_\psi \tag{4}$$

with $H : \Re_+ \times \Omega_\psi \to \Re^n$ where $\Omega_\psi \subset \Re^n$ is a non-empty open set.

Definition 1: [14] A solution $\psi(t)$ of the initial value problem (4) is maximal if it has no proper right extension that is also a solution of (4).

As an example, consider the initial value problem $\dot{\psi} = \psi^2$, $\psi(0) = 1$, whose solution is $\psi(t) = \frac{1}{1-t}$, $\forall t \in [0,1)$. The solution is maximal since it cannot be defined for $t > 1$. Stated otherwise, there is no proper extension of $\psi(t)$ to the right of $t = 1$ that is also a solution of the original initial value problem.

Theorem 1: [14] Consider the initial value problem (4). Assume that $H(t,\psi)$ is: a) locally Lipschitz on ψ for almost all $t \in \Re_+$, b) piecewise continuous on t for each fixed $\psi \in \Omega_\psi$ and c) locally integrable on t for each fixed $\psi \in \Omega_\psi$. Then, there exists a maximal solution $\psi(t)$ of (4) on the time interval $[0,\ t_{\max})$ with $t_{\max} > 0$ such that $\psi(t) \in \Omega_\psi$, $\forall t \in [0,\ t_{\max})$.

Proposition 1: [14] Assume that the hypotheses of Theorem 1 hold. For a maximal solution $\psi(t)$ on the time interval $[0,\ t_{\max})$ with $t_{\max} < \infty$ and for any compact set $\Omega'_\psi \subset \Omega_\psi$ there exists a time instant $t' \in [0,\ t_{\max})$ such that $\psi(t') \notin \Omega'_\psi$.

III. Control Design

Let us first define the joint position and velocity errors as:

$$e_q = [e_{q_1}, \ldots, e_{q_n}]^T$$
$$e_{\dot{q}} = [e_{\dot{q}_1}, \ldots, e_{\dot{q}_n}]^T$$

where $e_{q_i}(t) = q_i(t) - q_{d_i}(t)$ and $e_{\dot{q}_i}(t) = \dot{q}_i(t) - \dot{q}_{d_i}(t)$, $i = 1,\ldots,n$, as well as the stable linear filters:

$$s_i(e_{q_i}, e_{\dot{q}_i}) = \left(\tfrac{d}{dt} + \lambda\right) e_{q_i} \equiv e_{\dot{q}_i} + \lambda e_{q_i},\ i = 1,\ldots,n \tag{5}$$

with $\lambda > 0$, where s_i and e_{q_i} can be considered as the input and the output of the aforementioned stable linear filters respectively (i.e., $e_{q_i}(p) = \frac{s_i(p)}{p+\lambda}$ in the Laplace formulation with p denoting the Laplace frequency variable). The tracking control problem of (1) is equivalent to driving the joint position and velocity errors e_{q_i}, $e_{\dot{q}_i}$ on the surfaces $S_i = \{e_{q_i} \in \Re, e_{\dot{q}_i} \in \Re : s_i(e_{q_i}, e_{\dot{q}_i}) = 0\}$, $i = 1,\ldots,n$, since for $s_i(e_{q_i}, e_{\dot{q}_i}) = 0$, (5) represents a set of stable linear differential equations whose unique solution is $e_{q_i} = 0$, $e_{\dot{q}_i} = 0$. Hence, it can be reduced to the problem of driving the scalar quantities $s_i(e_{q_i}, e_{\dot{q}_i})$, $i = 1,\ldots,n$ to zero. Furthermore, bounds on $s_i(e_{q_i}, e_{\dot{q}_i})$ can also be directly translated into bounds on e_{q_i}, $i = 1,\ldots,n$. Thus, the scalar quantities $s_i(e_{q_i}, e_{\dot{q}_i})$, $i = 1,\ldots,n$ represent true measures of performance. More specifically, assume that $|s_i(e_{q_i}(t), e_{\dot{q}_i}(t))| < \rho_i(t)$, $\forall t \geq 0$ where $\rho_i(t) = (\rho_{i0} - \rho_\infty) e^{-lt} + \rho_\infty$ is an exponentially decaying performance function with appropriately selected parameters ρ_{i0}, ρ_∞, $l > 0$, $i = 1,\ldots,n$. The following proposition dictates how the parameters λ, l, ρ_∞ should be selected to guarantee exponential convergence of $e_{q_i}(t)$, $i = 1,\ldots,n$ in a prespecified arbitrarily small neighborhood of the origin with predefined rate, thus achieving tracking of $q_d(t)$ with prescribed transient and steady state performance.

Proposition 2: Consider the metrics $s_i(e_{q_i}(t), e_{\dot{q}_i}(t))$, $i = 1,\ldots,n$ as defined in (5) as well as the performance functions $\rho_i(t) = (\rho_{i0} - \rho_\infty) e^{-lt} + \rho_\infty$, $i = 1,\ldots,n$ with $l < \lambda$. If $|s_i(e_{q_i}(t), e_{\dot{q}_i}(t))| < \rho_i(t)$, $i = 1,\ldots,n$ for all $t \geq 0$ then all $e_{q_i}(t)$, $i = 1,\ldots,n$ converge at least e^{-lt} exponentially fast to the set $E = \left\{e \in \Re : |e| \leq \frac{\rho_\infty}{\lambda}\right\}$.

Proof: Consider the first order linear low pass filters with output $e_{q_i}(t)$ and pole $-\lambda$ driven by the scalar quantities $s_i(e_{q_i}(t), e_{\dot{q}_i}(t))$, $i = 1,\ldots,n$ (i.e., $s_i(t) = \left(\frac{d}{dt} + \lambda\right) e_{q_i}(t)$, $i = 1,\ldots,n$). It can be easily verified that:

$$|e_{q_i}(t)| \leq |e_{q_i}(0)| e^{-\lambda t} + \int_0^t e^{-\lambda(t-\tau)} |s_i(\tau)| d\tau$$

Employing $|s_i(e_{q_i}(t), e_{\dot{q}_i}(t))| < \rho_i(t) = (\rho_{i0} - \rho_\infty) e^{-lt} + \rho_\infty$, $\forall t \geq 0$ as well as $\lambda > l$, we obtain:

$$|e_{q_i}(t)| \leq \left(|e_{q_i}(0)| + \tfrac{\rho_{i0}-\rho_\infty}{\lambda - l}\right) e^{-lt} + \tfrac{\rho_\infty}{\lambda}, \forall t \geq 0,$$

which completes the proof. ■

In the sequel, we propose a state feedback control scheme $\tau(e_q, e_{\dot{q}}, t)$ for the robotic system (1) that guarantees $|s_i(e_{q_i}(t), e_{\dot{q}_i}(t))| < \rho_i(t)$, $i = 1,\ldots,n$ for all $t \geq 0$ and consequently, based on Proposition 2, tracking of the desired trajectory $q_d(t)$ with prescribed transient and steady state performance (i.e., all $e_{q_i}(t)$, $i = 1, \ldots, n$ converge at least e^{-lt} exponentially fast to the set $E = \left\{e \in \Re : |e| \leq \frac{\rho_\infty}{\lambda}\right\}$).

Theorem 2: Given the scalar quantities $s_i\left(e_{q_i}(t), e_{\dot{q}_i}(t)\right)$ defined in (5) and the required transient and steady state performance specifications, select λ and the exponentially decaying performance functions $\rho_i(t) = (\rho_{i0} - \rho_\infty) e^{-lt} + \rho_\infty$ such that: i) the desired performance specifications are met as described in Proposition 2 and ii) $\left|s_i\left(e_{q_i}(0), e_{\dot{q}_i}(0)\right)\right| < \rho_{i0}$. The state feedback control law $\tau(e_q, e_{\dot{q}}, t) = \left[\tau_1\left(e_{q_1}, e_{\dot{q}_1}, t\right), \ldots, \tau_n\left(e_{q_n}, e_{\dot{q}_n}, t\right)\right]^T$ with:

$$\tau_i\left(e_{q_i}, e_{\dot{q}_i}, t\right) = -k_i \ln\left(\frac{1 + \frac{s_i\left(e_{q_i}, e_{\dot{q}_i}\right)}{\rho_i(t)}}{1 - \frac{s_i\left(e_{q_i}, e_{\dot{q}_i}\right)}{\rho_i(t)}}\right), \ k_i > 0 \tag{6}$$

for $i = 1, \ldots, n$, guarantees tracking of the trajectory $q_d(t)$ with the desired transient and steady state performance specifications.

Proof: To prove our concept, we first define the normalized metric vector:

$$\xi_s(e_q, e_{\dot{q}}, t) = \left[\frac{s_1\left(e_{q_1}, e_{\dot{q}_1}\right)}{\rho_1(t)}, \ldots, \frac{s_n\left(e_{q_n}, e_{\dot{q}_n}\right)}{\rho_n(t)}\right]^T \tag{7}$$

as well as the generalized state vector $\xi = \left[q^T, \dot{q}^T, \xi_s^T\right]^T$ of the closed loop system. Differentiating ξ with respect to time and substituting the system dynamics (1) and the control input (6), the closed loop dynamical system of ξ may be written as:

$$\begin{aligned}\dot{\xi} &= h(t, \xi) \\ &= \begin{bmatrix} \dot{q} \\ -M^{-1}(q)\left(C(q, \dot{q})\dot{q} + G(q) + F_r(q, \dot{q}) - D(t) + K\ln\left(\frac{1+\xi_s}{1-\xi_s}\right)\right) \\ \rho^{-1}(t)\Big(-M^{-1}(q)\left(C(q, \dot{q})\dot{q} + G(q) + F_r(q, \dot{q}) - D(t)\right) \\ \quad -M^{-1}(q)K\ln\left(\frac{1+\xi_s}{1-\xi_s}\right) - \ddot{q}_d(t) + \lambda\left(\dot{q} - \dot{q}_d(t)\right) - \dot{\rho}(t)\xi_s\Big) \end{bmatrix}\end{aligned} \tag{8}$$

where $K = \mathsf{diag}([k_i])$ and $\rho(t) = \mathsf{diag}([\rho_i(t)])$. Let us also define the open set $\Omega_\xi = \Re^n \times \Re^n \times (-1, 1)^n$. In what follows, we proceed in two phases. First, the existence of a unique maximal solution $\xi(t)$ of (8) over the set Ω_ξ for a time interval $[0, t_{\max})$ (i.e., $\xi(t) \in \Omega_\xi$, $\forall t \in [0, t_{\max})$) is ensured. Then, we prove that the proposed control scheme (6) guarantees, for all $t \in [0, t_{\max})$: a) the boundedness of all closed loop signals of (8) as well as that b) $\xi(t)$ remains strictly within a compact subset of Ω_ξ, which leads by contradiction to $t_{\max} = \infty$ and consequently to the completion of the proof.

Phase A. The set Ω_ξ is nonempty and open. Moreover, the performance functions $\rho_i(t)$, $i = 1, \ldots, n$ have been selected to satisfy $\rho_i(0) > \left|s_i\left(e_{q_i}(0), e_{\dot{q}_i}(0)\right)\right|$, $i = 1, \ldots, n$. As a consequence, $\left|\frac{s_i\left(e_{q_i}(0), e_{\dot{q}_i}(0)\right)}{\rho_i(0)}\right| < 1$ which results in $\xi(0) \in \Omega_\xi$. Additionally, h is continuous on t and locally Lipschitz on ξ over the set Ω_ξ. Therefore, the hypotheses of Theorem 1 stated in Subsection II-B hold and the existence of a maximal solution $\xi(t)$ of (8) on a time interval $[0, t_{\max})$ such that $\xi(t) \in \Omega_\xi$, $\forall t \in [0, t_{\max})$ is ensured.

Phase B. We have proven in Phase A that $\xi(t) \in \Omega_\xi$, $\forall t \in [0, t_{\max})$ and more specifically that:

$$\frac{s_i\left(e_{q_i}(t), e_{\dot{q}_i}(t)\right)}{\rho_i(t)} \in (-1, 1), \ i = 1, \ldots, n \tag{9}$$

for all $t \in [0, t_{\max})$, from which we obtain that $s_i\left(e_{q_i}(t), e_{\dot{q}_i}(t)\right)$ is absolutely bounded by $\rho_i(t)$, $i = 1, \ldots, n$ for all $t \in [0, t_{\max})$. Hence, employing Proposition 2 we conclude that $\left|e_{q_i}(t)\right| \leq \left(\left|e_{q_i}(0)\right| + \frac{\rho_{i0} - \rho_\infty}{\lambda - l}\right) e^{-lt} + \frac{\rho_\infty}{\lambda}$, $i = 1, \ldots, n$ for all $t \in [0, t_{\max})$. Similarly, we may prove from (5), that $\left|e_{\dot{q}_i}(t)\right| \leq \left(\frac{2\lambda - l}{\lambda - l}(\rho_{i0} - \rho_\infty) + \lambda\left|e_{q_i}(0)\right|\right) e^{-lt} + 2\rho_\infty$, $i = 1, \ldots, n$ for all $t \in [0, t_{\max})$, which implies that $q(t) \in \Omega_q$ and $\dot{q} \in \Omega_{\dot{q}}$, $\forall t \in [0, t_{\max})$, with:

$$\Omega_q = \left\{q \in \Re^n : |q_i| \leq \bar{q}_i + \sup_{t \geq 0}\left\{q_{d_i}(t)\right\}, \ i = 1, \ldots, n\right\} \tag{10}$$

$$\Omega_{\dot{q}} = \left\{\dot{q} \in \Re^n : |\dot{q}_i| \leq \bar{\dot{q}}_i + \sup_{t \geq 0}\left\{\dot{q}_{d_i}(t)\right\}, \ i = 1, \ldots, n\right\} \tag{11}$$

with $\bar{q}_i = \left(\left|e_{q_i}(0)\right| + \frac{\rho_{i0} - \rho_\infty}{\lambda - l}\right) + \frac{\rho_\infty}{\lambda}$ and $\bar{\dot{q}}_i = \left(\frac{2\lambda - l}{\lambda - l}(\rho_{i0} - \rho_\infty) + \lambda\left|e_{q_i}(0)\right|\right) + 2\rho_\infty$, $i = 1, \ldots, n$. Furthermore, owing to (9), notice that the error signals:

$$e_{s_i}(t) = \ln\left(\frac{1 + \frac{s_i\left(e_{q_i}(t), e_{\dot{q}_i}(t)\right)}{\rho_i(t)}}{1 - \frac{s_i\left(e_{q_i}(t), e_{\dot{q}_i}(t)\right)}{\rho_i(t)}}\right), \ i = 1, \ldots, n \tag{12}$$

are well defined for all $t \in [0, t_{\max})$. Thus, differentiating with respect to time the positive definite and radially unbounded function $V_s = \frac{1}{2} e_s^T e_s$, where $e_s = \left[e_{s_1}, \ldots, e_{s_n}\right]^T$, we obtain:

$$\begin{aligned}\dot{V}_s = e_s^T r_s \big(&-M^{-1}(q)\left(C(q, \dot{q})\dot{q} + G(q) + F_r(q, \dot{q}) - D(t)\right) \\ &-M^{-1}(q) K e_s - \ddot{q}_d(t) + \lambda\left(\dot{q} - \dot{q}_d(t)\right) - \dot{\rho}(t)\xi_s\big).\end{aligned} \tag{13}$$

where $r_s = \mathsf{diag}\left(\left[\frac{2\rho_i(t)}{\rho_i^2(t) - s_i^2}\right]\right)$ is positive definite owing to the fact that $\lim_{t \to \infty} \rho_i(t) > 0$ and $\left|s_i\left(e_{q_i}(t), e_{\dot{q}_i}(t)\right)\right| < \rho_i(t)$, $i = 1, \ldots, n$ for all $t \in [0, t_{\max})$. Exploiting: i) the Extreme Value Theorem owing to the fact that $q(t) \in \Omega_q$, $\dot{q}(t) \in \Omega_{\dot{q}}$, $\forall t \in [0, t_{\max})$ and the vector fields $M(q)$, $C(q, \dot{q})$, $G(q)$, $F_r(q, \dot{q})$ are continuous and ii) the boundedness of $\dot{\rho}(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t)$ and $D(t)$, there exists a positive constant $\bar{F}$ independent of $t_{\max}$, such that:

$$\begin{aligned}\big\| -M^{-1}(q)&\left(C(q, \dot{q})\dot{q} + G(q) + F_r(q, \dot{q}) - D(t)\right) \\ &-\ddot{q}_d(t) + \lambda\left(\dot{q} - \dot{q}_d(t)\right) - \dot{\rho}(t)\xi_s\big\| \leq \bar{F}\end{aligned}$$

for all $t \in [0, t_{\max})$. Therefore, $\dot{V}_s < 0$ when $\left|e_{s_i}(t)\right| > \frac{\overline{m}\bar{F}}{\min\{k_j\}}$, $i = 1, \ldots, n$. Thus, we conclude that:

$$\left\|e_{s_i}(t)\right\| \leq \bar{e}_{s_i} = \max\left\{\left|e_{s_i}(0)\right|, \frac{\overline{m}\bar{F}}{\min\{k_j\}}\right\}, \ i = 1, \ldots, n \tag{14}$$

for all $t \in [0, t_{\max})$, which, by taking the inverse logarithmic function in (12), leads to:

$$-1 < \frac{e^{-\bar{e}_{s_i}}-1}{e^{-\bar{e}_{s_i}}+1} = \underline{\xi}_{s_i} \leq \frac{s_i\left(e_{q_i}(t), e_{\dot{q}_i}(t)\right)}{\rho_i(t)} \leq \bar{\xi}_{s_i} = \frac{e^{\bar{e}_{s_i}}-1}{e^{\bar{e}_{s_i}}+1} < 1 \tag{15}$$

for $i = 1, \ldots, n$ and for all $t \in [0, t_{\max})$. Moreover, the control input (6) remains bounded:

$$|\tau(e_q, e_{\dot{q}}, t)| \leq \tau^\star = \frac{\overline{m}\bar{F}\max\{k_j\}}{\min\{k_j\}}, \ \forall t \in [0, t_{\max}). \tag{16}$$

Up to this point, what remains to be shown is that $t_{\max}$ can be extended to ∞. In this direction, notice by (15) that $\xi(t) \in \Omega'_\xi$, $\forall t \in [0, t_{\max})$, where the set:

$$\Omega'_\xi = \Omega_q \times \Omega_{\dot{q}} \times \left[\frac{e^{-\bar{e}_s}-1}{e^{-\bar{e}_s}+1}, \frac{e^{\bar{e}_s}-1}{e^{\bar{e}_s}+1}\right]^n$$

is a nonempty and compact subset of Ω_ξ. Hence, assuming $t_{\max} < \infty$ and since $\Omega'_\xi \subset \Omega_\xi$, Proposition 1 in Subsection II-B dictates the existence of a time instant $t' \in [0, t_{\max})$ such that $\xi\left(t'\right) \notin \Omega'_\xi$, which is a clear contradiction. Therefore, $t_{\max} = \infty$. Thus, all closed loop signals remain bounded and moreover $\xi(t) \in \Omega'_\xi \subset \Omega_\xi$, $\forall t \geq 0$. Finally, from (15) we conclude:

$$\begin{aligned} -\rho_i(t) < \frac{e^{-\bar{e}_{s_i}}-1}{e^{-\bar{e}_{s_i}}+1}\rho_i(t) \leq \\ \leq s_i\left(e_{q_i}(t), e_{\dot{q}_i}(t)\right) \leq \\ \leq \frac{e^{\bar{e}_{s_i}}-1}{e^{\bar{e}_{s_i}}+1}\rho_i(t) < \rho_i(t) \end{aligned} \tag{17}$$

for $i = 1, \ldots, n$ and for all $t \geq 0$ and consequently, owing to Proposition 2, the solution of the tracking problem of system (1) with prescribed transient and steady state performance. ■

Remark 1: From the aforementioned proof, it is worth noticing that the proposed control scheme achieves its goals without residing on the need of rendering $\bar{e}_{s_i}$, $i = 1, \ldots, n$ (see (14)) arbitrarily small via increasing the control gains k_i, $i = 1, \ldots, n$. More specifically, notice that (15) and consequently (17), which encapsulates the prescribed performance notion, hold no matter how large the finite bounds $\bar{e}_{s_i}$, $i = 1, \ldots, n$ are. In the same spirit, the unknown system nonlinearities $M(q)$, $C(q, \dot{q})$, $G(q)$, $F_r(q, \dot{q})$ and the external disturbances $D(t)$ affect only the size of $\bar{e}_{s_i}$ but leave unaltered the achieved convergence properties as (17) dictates. In fact, the actual transient and steady state performance is determined by the selection of the performance functions $\rho_i(t)$, $i = 1, \ldots, n$ as well as of the parameter λ. Thus, contrary to what is common practice, the control gains k_i, $i = 1, \ldots, n$ should only be selected to yield reasonable control effort.

Remark 2: The dynamics of a robotic manipulator in the Cartesian workspace variables $p \in \Re^6$, involving the position and orientation of the end effector, can be expressed similarly with the joint representation as follows:

$$\tilde{M}(p)\ddot{p} + \tilde{C}(p, \dot{p})\dot{p} + \tilde{G}(p) + \tilde{F}_r(p, \dot{p}) = f + \tilde{D}(t)$$

where $\tilde{M}(p)$, $\tilde{C}(p, \dot{p})$, $\tilde{G}(p, \dot{p})$, $\tilde{F}_r(p, \dot{p})$ describe the effective nonlinearities of the manipulator (i.e., they represent the dynamics of the system as viewed by the Cartesian

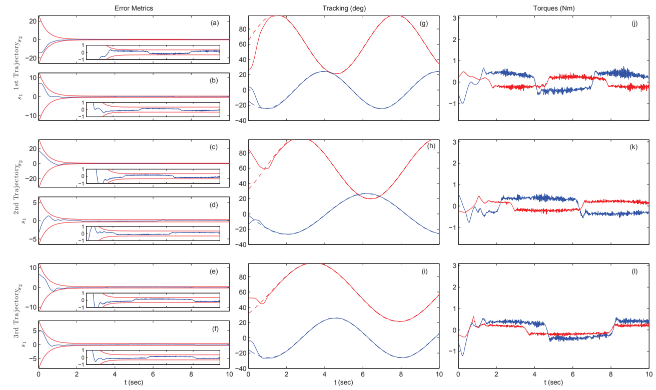

Fig. 1. Case A - Various Trajectories: Plots (a)-(f) depict the evolution of the error metrics s_1 and s_2 (blue lines) along with the imposed performance bounds (red lines). Details during the steady state are given in the subplots. Plots (g)-(i) visualize the tracking in q_1 (with blue color) and q_2 (with red color) respectively. The solid lines denote the actual response and the dashed lines the desired response. Plots (j)-(l) illustrate the required input torques.

workspace variables), $\tilde{D}(t)$ is an external disturbance term and f denotes the input force/torque at the end effector. It can be easily verified following similar arguments with the joint representation case that a desired trajectory $p_d(t) \in \Re^6$ (assuming it is far from the singular points of the manipulator) can be tracked with prescribed transient and steady state performance under the control scheme $f(e_p, e_{\dot{p}}, t) = [f_1(e_{p_1}, e_{\dot{p}_1}, t), \ldots, f_6(e_{p_6}, e_{\dot{p}_6}, t)]^T$ with:

$$f_i(e_{p_i}, e_{\dot{p}_i}, t) = -k_i \ln\left(\frac{1+\frac{s_i\left(e_{p_i}, e_{\dot{p}_i}\right)}{\rho_i(t)}}{1-\frac{s_i\left(e_{p_i}, e_{\dot{p}_i}\right)}{\rho_i(t)}}\right), \ k_i > 0$$

for $i = 1, \ldots, 6$, where:

$$\begin{aligned} e_p &= [e_{p_1}, \ldots, e_{p_6}]^T = p - p_d(t) \\ e_{\dot{p}} &= [e_{\dot{p}_1}, \ldots, e_{\dot{p}_6}]^T = \dot{p} - \dot{p}_d(t) \\ s_i &= e_{\dot{p}_i} + \lambda e_{p_i}, \ \lambda > 0, \ i = 1, \ldots, 6 \end{aligned}$$

and $\rho_i(t)$, $i = 1, \ldots, 6$ are appropriately selected performance functions.

IV. Experiments

To demonstrate the proposed model free control scheme, we performed two experimental studies in the joint workspace and the Cartesian workspace respectively, employing two joints (as a planar manipulator) of a Mitsubishi PA-10 robot arm. The robot servo controller communicates, via the ARCNET protocol, with a dedicated personal computer (PC) running a real-time operating system (Soft Real-Time Gentoo Linux) at the frequency of 100 Hz. It sends the current status of the robot arm (i.e., joint position and velocity) and receives torque commands, thus communicating with other PCs located in the same Local Area Network (LAN). The communication is achieved using a User Datagram Protocol (UDP) based scheme implemented in C/C++ as part of the overall control framework.

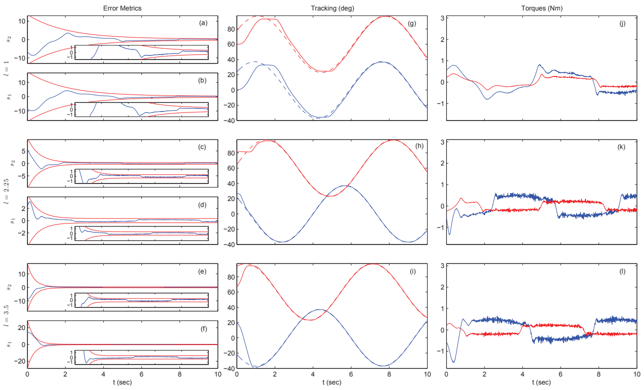

Fig. 2. Case A - Increasing l: Plots (a)-(f) depict the evolution of the error metrics s_1 and s_2 (blue lines) along with the imposed performance bounds (red lines). Details during the steady state are given in the subplots. Plots (g)-(i) visualize the tracking in q_1 (with blue color) and q_2 (with red color) respectively. The solid lines denote the actual response and the dashed lines the desired response. Plots (j)-(l) illustrate the required input torques.

Case A - Joint variables q_1 and q_2: The control target of this study is to track sinusoidal trajectories with various amplitude, frequency and phase of the form $q_{d_1}(t) = A_1 \sin(\omega_1 t + \phi_1)$, $q_{d_2}(t) = \frac{\pi}{3} + A_2 \sin(\omega_2 t + \phi_2)$ with A_i, ω_i, ϕ_i, $i = 1, 2$ randomly chosen in $[0.1\pi, 0.3\pi]$ rad, $[0.5, 1.0]$ rad/sec and $[0, 2\pi]$ rad respectively. Additionally, we required steady state errors of no more than 0.05 rad and minimum speed of convergence as obtained by the exponential e^{-3t}. Thus, following Proposition 2, we selected the parameter $\lambda = 20$ as well as the performance functions $\rho_i(t) = (\rho_{i0} - \rho_\infty) e^{-lt} + \rho_\infty$, $i = 1, 2$ with $l = 3$, $\rho_\infty = 1$ and $\rho_{10} > |s_1(e_{q_1}(0), e_{\dot{q}_1}(0))|$, $\rho_{20} > |s_2(e_{q_2}(0), e_{\dot{q}_2}(0))|$. Finally, we chose $k_1 = 0.5$ and $k_2 = 0.2$ to yield reasonable control effort. The results are illustrated in Fig. 1 for three different desired trajectories. More specifically, Fig. 1(a)-1(f) depict the evolution of the error metrics $s_1(e_{q_1}(t), e_{\dot{q}_1}(t))$, $s_2(e_{q_2}(t), e_{\dot{q}_2}(t))$ along with the imposed performance bounds by the selected functions $\rho_1(t)$ and $\rho_2(t)$. The tracking response is given in Fig. 1(g)-1(i) and the required input torques are illustrated in Fig. 1(j)-1(l). Furthermore, it should be noticed that the performance specifications were met for all three desired trajectories without altering the control gains k_1 and k_2. Additionally, to verify the fact that the achieved transient and steady state performance of the proposed control scheme is a priori specified by the appropriate selection of the performance functions $\rho_1(t)$ and $\rho_2(t)$, we increased the convergence rate (i.e., $l = 1 + 1.25j$, $j = 0, 1, 2$ with $\rho_\infty = 0.4$, see Fig. 2) and decreased the limit (i.e., $\rho_f = 1 - 0.325j$, $j = 0, 1, 2$ with $l = 3$, see Fig. 3) of the performance functions to achieve quicker response and more accurate tracking respectively. It should be also noted that the desired performance was achieved without altering the control gains k_1 and k_2. Finally, notice that as the response becomes quicker, the required input torques increase during the transient (as it was expected) while as ρ_∞ decreases and approaches the resolution (noise level) of the measurement device, undesirable effects (chattering) appear at the required input torques during the steady state.

Case B - Cartesian variables x and y: The control objective of this study is to track a 2D trajectory in the $x-y$ plane. The desired trajectory $x_d(t)$, $y_d(t)$ is a smooth representation of the acronym of our laboratory **CSL** (i.e., Control Systems Laboratory) with constant velocity (i.e., $\sqrt{\dot{x}_d^2(t) + \dot{y}_d^2(t)} =$ const). Additionally, we required steady state errors of no more than 0.01 m and minimum speed of convergence as obtained by the exponential e^{-t}. Thus, following Remark 2 and Proposition 2, we selected the parameter $\lambda = 20$ as well as the performance functions $\rho_i(t) = (\rho_{i0} - \rho_\infty) e^{-lt} + \rho_\infty$, $i \in \{x, y\}$ with $l = 1$, $\rho_\infty = 0.2$ and $\rho_{x0} > |s_x(e_x(0), e_{\dot{x}}(0))|$, $\rho_{y0} > |s_y(e_y(0), e_{\dot{y}}(0))|$. Finally, we chose $k_x = 0.3$ and $k_y = 0.3$ to yield reasonable control effort. The experimental results are illustrated in Figs. 4 and 5. More specifically, Fig. 4(a), 4(b) depict the evolution of the error metrics $s_x(e_x(t), e_{\dot{x}}(t))$, $s_y(e_y(t), e_{\dot{y}}(t))$ along with the imposed performance bounds by the selected functions $\rho_x(t)$ and $\rho_y(t)$. The tracking response and the trace of the end effector along with the desired trajectory are given in Fig. 4(c), 4(d) and 5 respectively while the required input torques are illustrated in Fig. 4(e). Furthermore, in order to visualize the trace of the end effector, we had attached a marker on it, that was writing on a planar surface. However, it should be noted that the motion of the marker was affected by significant tangential friction forces owing to the contact of the marker with the surface, which demonstrates the robustness of the proposed control scheme against external disturbances.

V. Video

This paper is accompanied by a short video demonstrating the efficiency of the proposed control scheme, using the Mitsubishi PA-10 robot arm in our laboratory. The first part of the video presents the experimental procedure in the joint workspace as described in Case A, for three different desired sinusoidal trajectories. It illustrates the ability of the controller to track considerably fast trajectories with prescribed transient and steady state performance despite the uncertainties of the robot dynamic model. The second part of the video demonstrates the experimental procedure in the Cartesian workspace as described in Case B. A red marker is attached at the end effector to enhance the visualization of its trace on a planar surface. It should be noticed that although significant tangential friction forces yielded, owing to the contact of the marker with the surface, they were compensated satisfactorily by the proposed control scheme. Finally, a more extensive HD video of the aforementioned experimental studies can be found at the following url:

`http://www.youtube.com/watch?v=TRsXkU2xMfo`

VI. Conclusions

This work proposes a robust model free controller for robotic manipulators that achieves tracking with prescribed transient and steady state performance. The developed controller exhibits the following important characteristics. First, the control scheme is of low complexity, it does not require any prior knowledge of the robot dynamic model and no estimation models are employed to acquire such knowledge. Furthermore, the a priori guaranteed performance simplifies significantly the selection of the controller parameters. Tuning of the controller gains is only confined to achieving reasonable control effort. Finally, as it is also demonstrated by the experimental studies, the controller can successfully compensate for significantly large external disturbances.

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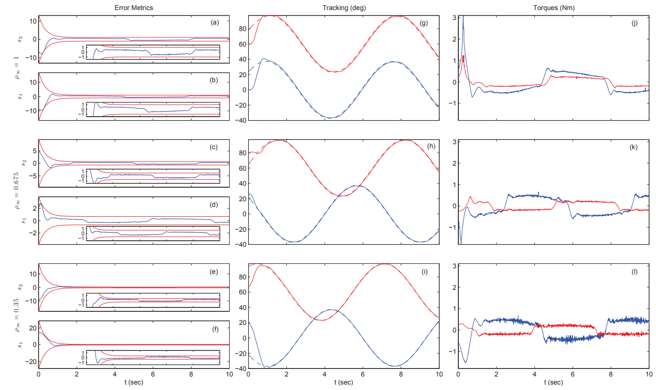

Fig. 3. Case A - Reducing ρ_∞: Plots (a)-(f) depict the evolution of the error metrics s_1 and s_2 (blue lines) along with the imposed performance bounds (red lines). Details during the steady state are given in the subplots. Plots (g)-(i) visualize the tracking in q_1 (with blue color) and q_2 (with red color) respectively. The solid lines denote the actual response and the dashed lines the desired response. Plots (j)-(l) illustrate the required input torques.

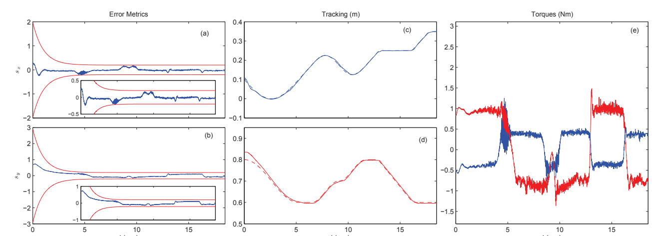

Fig. 4. Case B: Plots (a) and (b) depict the evolution of the error metrics s_x and s_y (blue lines) along with the imposed performance bounds (red lines). Details during the steady state are given in the subplots. Plots (c) and (d) visualize the tracking in x (with blue color) and y (with red color) axis respectively. The solid lines denote the actual response and the dashed lines the desired response. Plot (e) illustrates the required input torques.

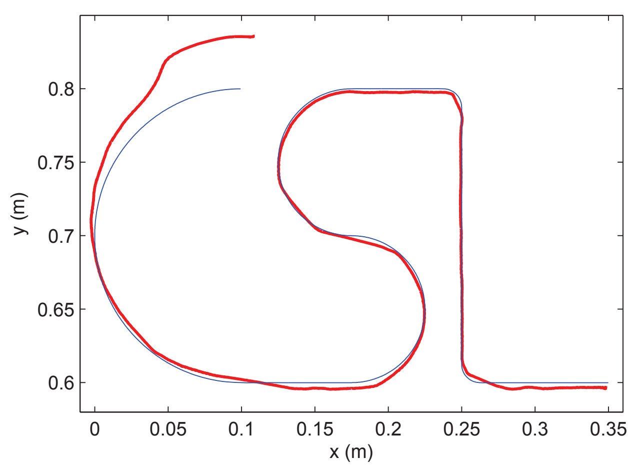

Fig. 5. The trace of the end effector on the planar surface (red line) along with the desired trajectory (blue line).